Theory of Computation
Homework 7.
Due Date: Thursday, November 5.

1. Convert the following CFG to CNF form. It has start variable $S$, terminal set $\{a, b, c\}$ and rules

$S \rightarrow AB \mid SX; A \rightarrow a \mid \lambda; B \rightarrow CA; C \rightarrow c \mid \lambda; X \rightarrow SAS.$

2. (a) Let $w \in \{a, b, c\}^*$. Define $\text{Sbst}(w, a, b)$ to be the string obtained by replacing all instances of the character $a$ in $w$ with $b$. e.g. $\text{Sbst}(ac,a,b) = bc$, $\text{Sbst}(cc,a,b) = cc$, $\text{Sbst}(abc,a,b) = bbc$. Let $L$ be a language over the alphabet $\{a, b, c\}$. Define $\text{Sbst}(L, a, b) = \{x \mid x = \text{Sbst}(w, a, b) \text{ for some } w \in L\}$.

Suppose that $L$ is a CFL. Show that $\text{Sbst}(L, a, b)$ is also a CFL by giving a CFG to generate $\text{Sbst}(L, a, b)$. Remember to explain why your solution is correct.

(b) Now define $\text{OneSubst}(w, a, b)$, or $\text{OS}(w, a, b)$ for short, to be the set of strings obtained by replacing one instance of the character $a$ from $w$ with a $b$. e.g. $\text{OS}(acacac,a,b) = \{bcacac, acbcac, acacbc\}$. Let $L$ be a language over the alphabet $\{a, b, c\}$. Define $\text{OS}(L, a, b) = \{x \mid x = \text{OS}(w, a, b) \text{ for some } w \in L\}$.

Suppose that $L$ is a CFL. Show that $\text{OS}(L, a, b)$ is also a CFL by giving a CFG to generate $\text{OS}(L, a, b)$. Remember to explain why your solution is correct.

3. (a) Let $E = \{a^ib^j \mid i < j\}$. Give a CFL to generate $E$.

(b) Let $F = \{a^ib^j \mid 2i > j\}$. Give a CFL to generate $F$.

(c) Let $I = \{a^ib^j \mid i < j < 2i\}$. Give a CFL to generate $I$.

Hint: Let $i = h + l$ and $j = h + 2l$. What can you say about $h$ and $l$?

4. Show that the following languages are not context free. Remember to give the full argument when using the Pumping Lemma, as shown in my handouts.

(a) $A = \{a^ib^jc^kb^jc^ka^l \mid i, j, k \geq 0\}$.

(b) $B = \{u\#v\#w \mid u, v, w \in \{a, b\}^*, \text{ the number of } a's \text{ in } u \text{ equals } |v|, \text{ and the number of } b's \text{ in } v \text{ equals } |w|\}$.

(c) $C = \{a^{2i} \mid i \geq 1\}$.

Comment. Any CFL over a 1-character alphabet is a regular language. I am not asking you to prove this and you may not use this fact.

(d) $D = \{x_1\#x_2\# \cdots \#x_l \mid x_h \in \{a, b\}^*, 1 \leq h \leq l, \text{ and for some } i, j, k, 1 \leq i < j < k, |x_i| = |x_j| = |x_k|\}$. 

1