1. Consider the set of directed, rooted, ordered binary trees in which any node may have 0, 1, or 2 children. These trees can be represented by expressions over the alphabet \{0, 1, 2\} as follows. The tree with a single vertex is represented by the expression 0. Any other tree is represented by starting with the digit corresponding to the number of children at the root node and then concatenating the expressions corresponding to the sub-trees for each child in order. All such trees of height 2 or less are shown below with their corresponding expressions.

(a) Give an inductive definition of the set \(T\) of expressions which represent such trees.

(b) Prove that \(T\) is freely generated. You will need to prove a lemma similar to the one proved in class (and in the solutions to last year’s problems).

(c) Prove that 1020 and 2210 are not in \(T\).
The height of such a tree is defined to be the longest possible path starting at the root node. For example, the tree $0$ has height $0$. The trees $10$ and $200$ have height $1$, and the rest of the trees shown above have height $2$. Give a recursive definition of a function which returns the height of any tree in $T$. 