1. Consider the Finite Automata shown in Figure 1.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{automata.png}
\caption{Automata for Problem 1.}
\end{figure}

Answer the following questions regarding each of the automata.

(a) Name its start vertex.

(b) List its final vertices.

(c) List the series of vertices the finite automata goes through as the following input is read: \textit{ababb}.

(d) Is \textit{ababb} recognized by the automata?

2. Consider the construction from Section 1.4 of the \textit{Finite Automata} handout. Show how to modify this construction so as to give a (Deterministic) Finite Automata \( M \) recognizing \( A \cap B \), assuming that \( A \) and \( B \) can both be recognized by Finite Automata. Explain briefly why your construction is correct; that is, explain why \( L(M) = A \cap B \).

3. Draw the graphs of (Deterministic) Finite Automata recognizing the following languages. In each subproblem, the alphabet being used is \( \Sigma = \{a, b\} \).
(a) The empty set Φ.
(b) The set containing just the empty string: {λ}.
(c) The set of all strings: Σ∗.
(d) The set of all strings having at least one character: \{ w \mid |w| \geq 1 \}.
(e) The set of all strings of length at two or more: \{ w \mid |w| \geq 2 \}.
(f) The set of all strings that begin with an a.
(g) The set of all strings that end with a b.
(h) The set of all strings containing aa as a substring.
(i) The set of all strings containing at least 4 a’s.
(j) The set of all strings either starting with an a and ending with a b, or starting with a b and ending with an a.
(k) The set of all strings such that no two b’s are adjacent.
(l) The set of all strings excepting aba: \{ w \mid w \neq aba \}.
(m) The set of all strings with alternating a’s and b’s.
(n) The set of all strings with a’s in the even positions.
(o) The set of all strings of even length whose second symbol is a b.
(p) The set of all strings containing at least two a’s.
(q) The set of all strings that contain aba as a substring.

4. Each of the following languages can be obtained by applying set operations (one of union, intersection, or complement) to simpler languages. By building (Deterministic) Finite Automata recognizing the simpler languages and then combining or modifying them, build (Deterministic) Finite Automata to recognize the following languages. In each subproblem, the alphabet being used is \( \Sigma = \{a, b\} \).

(a) The set of all strings containing at least one a and at least one b.
(b) The set of all strings containing at least one a and at most two b’s.
(c) The set of all strings with an even number of a’s and an odd number of b’s.
(d) The set of all strings with an even number of a’s and no adjacent a’s.
(e) The set of all strings that start with an a and end with a b.
(f) The set of all strings that have either 2 or 3 b’s and that have exactly 2 a’s.
(g) The set of strings in which all the a’s come before all the b’s.
(h) The set of all strings that do not contain the substring $aa$.

(i) The set of all strings other than the empty string: $\{w \mid w \neq \lambda\}$.

(j) The set of all strings other than $a$ or $bb$: $\{w \mid w \neq a, bb\}$.

(k) The set of all strings that include both an $a$ and a $b$. 