1. (a) Show that if $R \subseteq S$ then $R \cap \overline{S} = \emptyset$.
   (b) Let $\text{Reg-Contain} = \{\langle M_R, M_S \rangle \mid M_R$ and $M_S$ are DFAs recognizing regular languages $R$ and $S$ respectively, and $R \subseteq S\}$.
   Show that $\text{Reg-Contain}$ is decidable.
   Hint: Use a reduction to $\text{Empty-DFA}$.
   (c) Now let $\text{Reg-Contain-Sym} = \{\langle M_R, M_S \rangle \mid M_R$ and $M_S$ are DFAs recognizing regular languages $R$ and $S$ respectively, and either $R \subseteq S$ or $S \subseteq R\}$.
   Show that $\text{Reg-Contain-Sym}$ is decidable.

2. Let $\text{Reg-Rev} = \{\langle M \rangle \mid M$ is a DFA and for each $w \in \Sigma^*$, where $\Sigma$ is the input alphabet for $M$, if $M$ recognizes $w$ then $M$ also recognizes $w^R\}$.
   Show that $\text{Reg-Rev}$ is decidable. You may assume the result of Problem 4, Homework 3.
   Hint. Let $M^R$ be a DFA recognizing $(L(M))^R$: $M^R$ recognizes the reversal of strings recognized by $M$. If $M \in \text{Reg-Rev}$, what is the relationship between $L(M)$ and $L(M^R)$? Your decision procedure needs to test whether this relationship holds.

3. Let $\text{Reg-No-Ext} = \{\langle M \rangle \mid M$ is a DFA such that if $w$ is a string recognized by $M$, then no extension of $w$, no string $wx$ with $|x| \geq 1$, is recognized by $M\}$. Show that $\text{Reg-No-Ext}$ is decidable.
   Hint. What can you say about $M$’s graph if the strings it recognizes satisfy the no-extension property? Specifically, suppose that $w \in L(M)$ and the $w$-recognizing path ends at final vertex $f$. What can and cannot be reached from $f$?

4. Let $\text{CFL-Int-a-Star} = \{\langle G \rangle \mid G$ is a context free grammar, and $L(M)$, the language it generates, satisfies $L(M) \cap a^* \neq \emptyset\}$. Show that $\text{CFL-Int-a-Star}$ is decidable.
   Hint. Use a reduction to $\text{Empty-CFL}$.