1. (a) Let \( w \in \{a, b, c\}^* \). Define \( \text{Remove-c}(w) \) to be the string obtained by deleting all instances of the character \( c \) from \( w \). e.g. \( \text{Remove-c}(ab) = ab \), \( \text{Remove-c}(cc) = \lambda \), \( \text{Remove-c}(abc) = ab \), \( \text{Remove-c}(acacac) = aaa \).

Let \( L \) be a language over the alphabet \( \{a, b, c\} \). Define \( \text{Remove-c}(L) = \{x \mid x = \text{Remove-c}(w) \text{ for some } w \in L\} \).

Suppose that \( L \) is a CFL. Show that \( \text{Remove-c}(L) \) is also a CFL by giving a CFG to generate \( \text{Remove-c}(L) \).

(b) Now define \( \text{Remove-One-c}(w) \) to be the set of strings obtained by deleting one instances of the character \( c \) from \( w \). e.g. \( \text{Remove-One-c}(acacac) = \{aacac, acaac, acaca\} \).

Let \( L \) be a language over the alphabet \( \{a, b, c\} \). Define \( \text{Remove-One-c}(L) = \{x \mid x = \text{Remove-One-c}(w) \text{ for some } w \in L\} \).

Suppose that \( L \) is a CFL. Show that \( \text{Remove-One-c}(L) \) is also a CFL by giving a CFG to generate \( \text{Remove-One-c}(L) \).

2. (a) Let \( E = \{a^i b^j \mid i < j\} \). Give a CFL to generate \( E \).

(b) Let \( F = \{a^i b^j \mid i > 2j\} \). Give a CFL to generate \( F \).

(c) Let \( I = \{a^i b^j \mid j < i < 2j\} \). Give a CFL to generate \( I \).

3. Show that the following languages are not context free. Remember to give the full argument when using the Pumping Lemma, as shown in my handouts.

(a) \( A = \{a^m b^n c^m d^n \mid m, n \geq 0\} \).

(b) \( B = \{w \mid w \in \{a, b, c\}^* \text{ and the number of } a's, b's \text{ and } c's \text{ in } w \text{ are all equal}\} \).

(c) \( C = \{a^{2i} \mid i \geq 0\} \).

Comment. Any CFL over a 1-character alphabet is a regular language. I am not asking you to prove this and you may not use this fact.

(d) \( D = \{x_1 \# x_2 \# \cdots \# x_k \mid x_h \in \{a, b\}^*, 1 \leq h \leq k, \text{ and for some } i, j, 1 \leq i < j \leq k, x_i = x_j\} \).