1. Give CFG’s to generate the following languages.

(a) \( A = \{ w \mid w \in \{a, b\}^* \text{ and } w = w^R \}. \) \( A \) is the language of palindromes, strings that read the same forward and backward.  
   Hint: Be sure to handle strings of all possible lengths.

(b) \( B = \{ w\#x \mid w, x \in \{a, b\}^* \text{ and } w^R \text{ is an initial substring of } x \}. \)  
   Hint: \( x \) can be written as \( x = w^Ry \) for some \( x \in \{a, b\}^* \).

(c) \( C = \{ w \mid w \in \{a, b\}^* \text{ and } w \text{ contains an equal number of } a \text{'s and } b \text{'s} \}. \)  
   Hint: suppose that the first character in \( w \) is an \( a \). Let \( x \) be the shortest initial substring of \( w \) having an equal number of \( a \text{'s and } b \text{'s}. \) If \( |x| < |w| \), then \( w \) can be written as \( w = xy \); what can you say about \( y \)? Otherwise, \( x = w \) and \( w \) can be written as \( w = azb \); what can you say about \( z \)?

2. Let \( A \) be a CFL generated by a CFG \( G_A \). Give a CFG grammar \( G_{A^*} \), based on \( G_A \), to generate \( A^* \). Argue that \( L(G_{A^*}) = A^* \).

3. Let \( A \) be a CFL generated by a CFG \( G_A \) with start symbol \( S_A \). Consider adding the rule \( S_A \rightarrow S_A S_A \) to the grammar \( G_A \), and let \( B \) be the language generated by the changed grammar. Give an example language \( A \) for which \( B \neq A^* \).

4. Convert the following CFG to CNF form. It has start variable \( S \), terminal set \( \{a, b, c\} \) and rules  
   \[ S \rightarrow SBS \mid BC; \ B \rightarrow ab \mid \lambda; \ C \rightarrow c \mid \lambda. \]