1. Draw the graphs of NFAs recognizing the following languages.
   
   (a) $L = a^*$, using a 1-vertex NFA.
   
   (b) $L = \{w \mid w \text{ has } aa \text{ as a substring}\}$, using a 3-vertex NFA.
   
   (c) $L = \{w \mid w \text{ ends with } bb\}$, using a 3-vertex NFA.
   
   (d) $L = \{w \mid w \text{ is of even length or the second symbol in } w \text{ is a } b \text{ (or both)}\}$, using a 5-vertex NFA.

2. Using the methods of Section 1.1 in *Finite Automata, Part 2*, give the graphs of NFAs that recognize the following languages.
   
   (a) $A \cup B$, where $A = \{w \mid w \text{ begins with an } a\}$, $B = \{x \mid x \text{ ends with a } b\}$, and $A, B \subseteq \{a, b\}^*$.
   
   (b) $C \circ D$ where $C = \{w \mid |w| \geq 2\}$, $D = \{x \mid x \text{ contains } aa \text{ as a substring}\}$, and $C, D \subseteq \{a, b\}^*$.
   
   (c) $E^*$, where $E = \{w \mid \text{all characters in even positions in } w \text{ are } a's\}$, and $E \subseteq \{a, b\}^*$.
   
   (d) $F = \Phi^*$ (recall that $\Phi$ is the empty language). Trust the construction. What strings, if any, are in $\Phi^*$?

3. (a) Construct an NFA recognizing the language $L = \{ba, bab\}^*$.

   (b) Convert this NFA to a DFA recognizing the same language using the method of Section 1.2 of *Finite Automata, Part 2*. You need show only the portion of the DFA reachable from the start vertex.

4. Let $L$ be a regular language. Define the reverse of $L$, $L^R = \{w \mid w^R \in L\}$, i.e. $L^R$ contains the reverse of strings in $L$ (for $(x^R)^R = x$ for any string $x$). Show that $L^R$ is also regular.

   Hint. Suppose that $M$ is a DFA (or an NFA if you prefer) recognizing $L$. Construct an NFA $M^R$ that recognizes $L^R$; $M^R$ will be based on $M$. Remember to argue that $L(M^R) = L^R$. 