1. Give a deduction (from $\emptyset$) of ($\forall x \phi$) $\rightarrow$ $\exists x \phi$.

2. (Re-replacement lemma)
   
   (a) Show by example that $(\phi^x_y)^y$ is not in general equal to $\phi$. Show that it is possible both for $x$ to occur in $(\phi^x_y)^y$ at a place where it does not occur in $\phi$, and for $x$ to occur in $\phi$ at a place where it does not occur in $(\phi^x_y)^y$.
   
   (b) Show that if $y$ does not occur at all in $\phi$, then $x$ is substitutable for $y$ in $\phi^x_y$ and $(\phi^x_y)^y = \phi$.
   
   Suggestion: use induction on $\phi$.

3. Show that the transitivity of equality follows from the axioms, that is,
   
   $\vdash \forall x \forall y \forall z (x = y \rightarrow (y = z \rightarrow x = z))$.

4. Prove that Axiom Groups 3 and 4 are valid.

5. Prove the equivalence of the following two statements.
   
   (a) If $\Gamma \models \phi$, then $\Gamma \vdash \phi$.
   
   (b) Any consistent set of formulas is satisfiable.

6. Let $\Gamma = \{-\forall v_1 P v_1, P v_1, P v_2, P v_3, \ldots\}$. Is $\Gamma$ consistent? Is $\Gamma$ satisfiable?