1. Let \( w \in \{a, b, c\}^* \). Define \( \text{Remove-c}(w) \) to be the string obtained by deleting all instances of the character \( c \) from \( w \). e.g. \( \text{Remove-c}(ab) = ab \), \( \text{Remove-c}(cc) = \epsilon \), \( \text{Remove-c}(abc) = ab \), \( \text{Remove-c}(acacac) = aaa \).

Let \( L \) be a language over the alphabet \( \{a, b, c\} \). Define \( \text{Remove-c}(L) = \{x | x = \text{Remove-c}(w) \text{ for some } w \in L\} \).

a. Suppose that \( L \) is a CFL. Show that \( \text{Remove-c}(L) \) is also a CFL by giving a CFG to generate \( \text{Remove-c}(L) \).

b. Now suppose \( L \) is recognized by a pda \( M \). Give a pda \( \tilde{M} \) to recognize \( \text{Remove-c}(L) \).

Comment: The two parts are equivalent; nonetheless, I am asking for a separate construction for each part.

2. Let \( A \) and \( B \) be CFLs. Show that \( A \cup B \), \( A \circ B \), \( A^* \) are also CFLs, by giving CFGs to generate them.

3.a. Let \( C = \{uav\#xby | u, v, x, y \in \{a, b\}^* \text{ and } (|u| - |v|) = (|x| - |y|)\} \). Give a pda to recognize \( C \) or a CFL to generate \( C \).

b. Let \( D = \{w\#z | w, z \in \{a, b\}^* \text{ and } |w| \neq |z|\} \). Give a pda to recognize \( D \), or a CFG to generate \( D \).

c. Show that \( C \cup D = \{s\#t | s, t \in \{a, b\}^* \text{ and } s \neq t\} \).

4. a. Let \( E = \{a^ib^j | i < j\} \). Give a CFL to generate \( E \).

b. Let \( F = \{a^ib^j | i > 2j\} \). Give a CFL to generate \( F \).

c. Let \( I = \{a^ib^j | j < i < 2j\} \). Give a CFL to generate \( I \).