1. Let $G = (V, E)$ be a directed graph with non-negative edge lengths and let $s$ be a designated vertex. Using Dijkstra’s algorithm as a subroutine, give an efficient algorithm to compute for each $v \in V$ the length of a shortest path from $s$ to $v$ and back to $s$.

2. Suppose that you were given an efficient algorithm for the Hamiltonian Circuit problem. Using it as a subroutine, give an efficient algorithm to solve the Hamiltonian Path problem. Note your algorithm needs to proceed as follows. Given input $(G, s, t)$ to the Hamiltonian Path problem, your algorithm needs to construct a new graph $H$, then run the Hamiltonian Circuit algorithm on $H$, and finally take the answer just computed by the Hamiltonian Circuit algorithm and use it to determine the answer to the Hamiltonian Path problem for $(G, s, t)$.

3. The Traveling Salesman problem is the following problem.
Input: A directed graph $G$ with integer edges lengths, and an integer bound $b$.
Task: Determine whether $G$ has a Hamiltonian Circuit of length at most $b$.

Suppose that you were given an efficient algorithm for the Traveling Salesman problem. Using it as a subroutine, give an efficient algorithm for the Hamiltonian Circuit problem.

4. Let $\text{INF}_{PDA} = \{ \langle A \rangle \mid A$ is a PDA and $L(A)$ has infinite size $\}$. Show that $\text{INF}_{PDA}$ is computable. Your task is to give an algorithm to determine if $L(A)$ has infinite size given input $\langle A \rangle$.


5. 4.19 (Sipser, both editions). For those with the international edition, note that the problem mentions: $M$ is a DFA that accepts $w^R$ whenever it accepts $w$.
Hint. You need to give an algorithm to recognize this language. You will want to use a reduction, that is a subroutine for a suitable problem for which we have already seen an algorithm. I suggest using the algorithm for $EQ_{DFA}$, which is the language of pairs of DFAs $(M_A, M_B)$ that accept the same language.