Consider the case that $d \in \text{CHAIN}$ and assume that $d$ satisfies Property $\chi$. Thus, we let us present a heuristic by which we can systematically derive the auxiliary transition relation. By premise $\text{CHAIN}$, all states beyond $d$ satisfy $\chi$, and the transition relation can be written as $\chi^{-}$. In particular, $(\chi)_{\chi^{-}} = (\chi)_{\chi^{-}} = (\chi)_{\chi^{-}}$. Thus, we can summarize premises $C_{2}$ and $C_{3}$ for the case $i = 1$. From premises $C_{2}$ and $C_{3}$, we can infer $\chi^{-}$, and hence, $\chi$.

To prove property $\text{CHAIN}$, in order to prove Property $\chi$, we construct the case that $d \in \text{CHAIN}$ and $d$ satisfies Property $\chi$. Thus, we can systematically derive the auxiliary transition relation. By premise $\text{CHAIN}$, all states beyond $d$ satisfy $\chi$, and the transition relation can be written as $\chi^{-}$. In particular, $(\chi)_{\chi^{-}} = (\chi)_{\chi^{-}} = (\chi)_{\chi^{-}}$. Thus, we can summarize premises $C_{2}$ and $C_{3}$ for the case $i = 1$. From premises $C_{2}$ and $C_{3}$, we can infer $\chi^{-}$, and hence, $\chi$.
In the computation of this table, we made free use of the relevant invariants which can be viewed as an inequality with the unknown $h_2$. For a given $f_2$, we can try to solve such an inequality by coming the iteration sequence

\[
\begin{array}{c}
\text{Imp}\\(0 \land f_2) \text{Imp} \\
1 : h_2
\end{array}
\]

until it converges.

\[
\text{...}
\]

\[
\begin{array}{c}
(0 \land f_2) \text{Imp} \\
1 : h_2
\end{array}
\]

We obtain the iteration sequence for the case that $\nu f_2 = h_2$ and $\nu f_2 = 0$.

Continued

Computation of $f_1$ Contindued