Homework 2

Professor: Dennis Shasha

Due: Tuesday, November 13

Each question is worth 10 points. You may work with one partner and sign both of your names to your paper. You will receive the same grade. There are a maximum of 60 points, so please hand in only six problems. Doing more problems will not help as I will read only the first six I receive.

**Question 1**

Prove that the spanning tree algorithm that begins the discussion on failure-free connected graphs (which continues with termination, broadcast, snapshots, and synchronizers) does indeed find a spanning tree, given that the network is connected and no failures occur. That is, show that there are no cycles and that there is a path from every node to the root.

**Question 2**

Can three phase commit produce an inconsistent outcome if timeouts are not reliable but the network is completely connected and messages arrive eventually? Timeouts are not reliable means that a site may send a message such as an “Are you awake?” message to another site, receive no reply, yet the other site may still be up and the network is fine.

**Question 3**

Show that the snapshot protocol is correct even if the messages sent during a round are not from nearest neighboring sites. Correctness means AVOIDING the following: A message is sent from A to B (not necessarily neighbors) and the message is sent by A after A reaches its snapshot for a round but is received by B before B reaches its snapshot for that same round. Snapshot messages still go to direct neighbors, but it may be that data messages pass through several nodes (e.g. all the way from A to B in a single round).

**Question 4**

Determine the worst case work complexity of LeLann’s real-time ethernet algorithm as a function of the number of attempting sites $A$ in a network with $2^k$ nodes where $2^{k-1} \leq A \leq 2^k$. By work, I mean the number of attempted message transmissions from all sites (some of which will be successful and others will collide) put together from the first duel to the last successful transmission of the group. Another measure would be the total waiting time, but let’s stick with attempted message transmissions. Each site can be identified by a $k$ bit number.
Question 5

Competitive analysis

Consider the following puzzle: you have 90 tickets that you can trade for payoff cards which you will be offered over the next two hours. You know initially that the cards will have be worth either $1 or $5, but you don’t know how many total cards you will be offered, the percentage of either kind, or even when the giver will stop giving you cards. Further, if you have tickets after the giver stops, they are worth nothing.

You want to compare your strategy to that of a clairvoyant (can see the future) adversary. You want to ensure that the adversary won’t get more than twice as much as you do (less if possible). Assume that you are offered the same possibilities as the adversary. Can you find a strategy that will guarantee that? What would be your strategy be if the big tickets are worth $1 million instead of $5?

Question 6

There are six people arranged in a line in a warehouse (in general n, but let’s start with six). The exit is at the left end of the warehouse one minute to the left of position 1.

1 ——– 2 ————3 ....

It takes one minute to walk from one position to the next position down. Demand can come to any numbered place along the line. The question is what should the worker at position i do when an order for a good arrives?

Zone model: Person i walks the good to the exit.

Bucket brigade model: Walk left until you run into someone either standing still or coming the other way and then give the package to that person. Transferring the package takes zero time. If there is a tie, a package in movement takes precedence over a package that has just been ordered (demanded).

After delivery, walk back to your station unless you’re handed a package. (Never walk beyond your station.)

Warm-Up: Suppose one package request arrives at each of the six stations. Would things be better if we used zone or bucket brigade?

Solution to warm-up: even though the bucket brigade model entails far more handing off of packages, at every time period, every package is moving towards the exit (assuming zero transfer cost). So the two models have the same delivery time.

A. Is there any situation in which bucket brigade is slower? If so, ever by as much as a factor of two? (By slower, I mean time for total task. The total task begins when the first order arrives and ends when the last package leaves the exit.)

B. Is there any situation in which bucket brigade is faster? If so, ever by as much as a factor of two?

Question 7

Baskerhound at the Post Office.

“Imagine that a network works like the paper mail service (sometimes called snail mail),” said the chief detective. “Most letters arrive in one day. However, not all do.
Some take many days, though they will arrive eventually. The letters may not arrive in order. That is, if A sends a letter to B on day 1 and then another letter to B on day 2, the first letter may arrive last.

“Now, there are 17 people, each of whom has a single message that he or she must send to the other 16 people. At the end, everyone should have the messages from themselves and everyone else. Each letter can contain only one message. All letters must go through a single central post office. Each person has 17 envelopes (one more than may seem strictly necessary up to this point).

“All would be well except that Baskerhound, our adversary, may try to read these messages. He may look at all the letters in the post office at one, perhaps two points of time that he can choose. He won’t alter the letters, but he may read them.

“We want to avoid letting Baskerhound know the messages of all 17 people, though he may know some of them.”

“So you have to make sure that copies of all the messages aren’t in the mail at the same time?” Ecco asked.

“Exactly,” responded the chief detective. “Here is how the solution might go if there were just two people, Bob and Alice and Baskerhound could just look once at the post office. Bob sends to Alice; when she receives, she sends her message to Bob.”

“Can we assume that some participant, say number 1, starts the protocol?” asked Ecco.

“Yes, assume that 1 knows when the protocol is to begin and is the only one to send the first day (though not necessarily to everyone),” responded the chief detective.

“Finally,” Ecco continued, “can a participant send any other message in a letter, e.g. ‘I got a message from these participants’?”

“The only other message they can send is the simple phrase ‘all clear,’ ” responded the chief detective.

Answer the following questions:

a. Baskerhound can look in the post office only once. We want a solution that will take two or fewer days if all letters arrive in a day, but which doesn’t allow Baskerhound to see all the messages no matter how long any given letter takes. Remember that each participant has only 17 envelopes and one message is allowed per envelope.¹ (Actually it can be done with 16 envelopes.)

b. Baskerhound can look twice. Try to find a solution that will take only five days if all letters arrive in a day, but which doesn’t allow Baskerhound to see all the messages in his two looks no matter how long any given letter takes.

c. Baskerhound can look twice and only needs 10 distinct messages (i.e. the messages from 10 different people) to do severe damage. Each participant has 35 envelopes. Try to find a solution that takes eight days, if all letters arrive in a day, but which doesn’t allow Baskerhound to see all the messages in his two looks no matter how long any given letter takes.

¹So suppose that each letter arrives in a day, then the protocol should guarantee that everything finishes in two days. Suppose some letters take 0 days, others take 13 days, others take 9 days. Then there is no performance guarantee but Baskerhound should still not be able to see all messages.
Question 8

Solve the following puzzles and state the knowledge of each of the players at each point in the protocol. That is, show the state of knowledge of each participant that allows that participant to reach a conclusion.

Consider the following game: shuffle a deck of cards, each person takes a card, and holds it to his or her forehead. So each player sees all cards except his or her own. The best card wins. Ace is high and suits don’t matter. So ties are possible.

This puzzle has to do with inferring what players have hearing only what they say. To be concrete, suppose that Jordan, David, and Caroline each pick a card. Caroline speaks first, then David, and then Jordan. Each player says either:
"I win. (I have a higher card than anyone else.)"
"I lose. (Someone else has a higher card than I have.)"
"I tie as a winner. (I have the highest card, but someone else may have a card of equal value.)"
"I don’t win, but may tie or lose."
"I don’t lose, but may tie or win."
"I don’t know."

The players are assumed to be perfect logicians, but reveal information only through one of the phrases above. They say the strongest statement they know.

Warm-up: Suppose Caroline says "I don’t know.” Then David says "I lose,” then Jordan says "I lose." What do we know about the cards?

Solution to Warm-up: Caroline must have an ace and the others have less than an ace. Here’s why. Caroline cannot see an ace, otherwise she could say "I will not win”. So, we have kc "There is no other ace around”. Because she says she doesn’t know, David realizes that he doesn’t have an ace. kd "I don’t have an ace”. If he sees an ace on Caroline’s forehead, then he knows he loses. There is no other reason for David to know he will lose. Similarly for Jordan.

Here are some questions. See what you can infer: 1. Caroline says "I don’t know.” David says "I don’t win.” Then Jordan says "I win" 2. Caroline says "I don’t know.” David says "I don’t win.” Then Jordan says "I tie" 3. Caroline says "I don’t know.” David says "I don’t win.” Then Jordan says "I don’t win" 4. Caroline says "I don’t know.” David says "I don’t know.” Then Jordan says "I don’t know.” Then (having heard this) Caroline says "I lose.” What do David and Jordan then say and what do you know about the cards?