Theory of Computation
Homework 14.

Due Date: Tuesday, December 12.

1. A *Dominating Set* $D$ for an undirected graph $G = (V, E)$ is a subset of the vertices such that each vertex $v \in V$ is either in the Dominating Set or adjacent to some vertex in the Dominating Set. Let Dominating Set, or DS for short, be the following language:

$$DS = \{(G, k) \mid G \text{ has a dominating set containing } k \text{ vertices}\}.$$  

Show that DS is NP-Complete, using a reduction from Vertex Cover (VC for short).

Here the task is as follows. Given a pair $(H, h)$, an input for the VC problem, you need to show how to construct a pair $(G, k)$, an input for the DS problem, such that:

$$(H, h) \in \text{Vertex Cover} \iff (G, k) \in \text{DS}.$$  

2. The Timetable Problem. The input has several parts: an integer $t$, a list $F_1, F_2, \ldots, F_k$ of $k$ final exams to schedule, a list of students $S_1, S_2, \ldots, S_n$; in addition, each student is taking some subset of exams, specified in a list $SL_i$ for student $i$, $1 \leq i \leq n$. The task is to schedule the exams so that a student is scheduled for at most one exam in any given time slot. The problem is to determine if there is a schedule using only $t$ time slots.

Show that the Timetable Problem is NP-Complete.

I advise reducing 3-Color to the Timetable Problem (see Problem 4 below). That is, you have the following task. Given a graph $G = (V, E)$, an input to the 3-color problem, you need to create an input $T$ for the Timetable problem, such that $G$ is 3-colorable, if and only if $T$ is schedulable. The main issues in the construction are to decide what in the timetable corresponds to a vertex of $G$ and what corresponds to an edge. Don’t forget to choose a suitable value for $t$.

3. Sipser text, no. 7.28 (2nd edition), 7.27 (1st edition). You may assume $\neq$-assignment is NP-Complete (see problem 7.24 (2nd ed.), 7.22 (1st ed)).