

Honors Algorithms — G22.3520-010 Fall 2005 — Problem Set 7
For study purposes only — Do not hand in

1. (a) Explain why the class of context-free languages is closed under the regular operations of union, concatenation, and star.
(b) From part (a), show that any regular language is context free.
(c) Show that the class of context-free languages is not closed under intersection or complement.
(d) Show that the intersection of a context-free language with a regular language is context free.
2. For $w \in \{0, 1\}^*$, let $N_0(w)$ (resp., $N_1(w)$) be the number of times 0 (resp., 1) appears in w .
(a) Let $A = \{w \in \{0, 1\}^* : N_0(w) = 2N_1(w)\}$. Give a CFG for A , and a careful proof that your grammar is correct.
(b) Let $B = \{w \in \{0, 1\}^* : 0.4N_1(w) \leq N_0(w) \leq N_1(w)\}$. Give a PDA for B .
(c) Let $C = \{x\#y : x, y \in \{0, 1\}^*, x \neq y\}$. Show that C is context free.
3. (a) Let A be the language of all strings over $\{a, b, c, d\}$ such that the number of a 's equals the number of b 's, and the number of c 's equals the number of d 's. Show that A is not context free.
(b) Let $B = \{0^{nm}1^m : n, m \in \mathbb{Z}_{\geq 0}\}$. Show that B is not context free.
4. Sipser 7.22 (this is a solved exercise, but you might try it first before reading the solution)
5. Sipser 7.26.
6. An AND formula is a Boolean formula $\phi = c_1 \wedge \cdots \wedge c_k$, where each c_i an "AND clause" of the form $(\alpha_i \wedge \beta_i \wedge \gamma_i)$ for literals $\alpha_i, \beta_i, \gamma_i$ (and a literal is either a variable or its negation). For $\ell = 0, \dots, k$, we say that ϕ is ℓ -satisfiable there is some assignment of truth values to variables that simultaneously satisfies at least ℓ AND clauses. Let

$$\text{MaxAND} = \{\langle \phi, \ell \rangle : \phi \text{ is an } \ell\text{-satisfiable AND formula}\}.$$

Show that *MaxAND* is NP-complete.

7. Let

$$\text{SetPACK} = \{\langle C, k \rangle : C \text{ is a collection of finite sets, at least } k \text{ of which are disjoint}\}.$$

Show that *SetPACK* is NP-complete.